Formal Description of Autopoiesis Based on the Theory of Category

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Abstract. Since the concept of autopoiesis was proposed as a model of minimal living systems by Maturana and Varela, there has been still few mathematically strict models to represent the characteristics of it because of its difficulty for interpretation. This paper proposes a formal description of autopoiesis based on the theory of category and Rosen's perspective of "closure under efficient cause".

1 Introduction

Autopoiesis gives a framework in which a system exists as an organism through physical and chemical processes, based on the assumption that organisms are machinary [3]. According to the original definition of it by Maturana and Varela, an autopoietic system is one that continuously produces the components that specify it, while at the same time realizing itself to be a concrete unity in space and time; this makes the network of production of components possible. An autopoietic system is organized as a network of processes of production of components, where these components:

1. continuously regenerate and realize the network that produces them, and
2. constitute the system as a distinguishable unity in the domain in which they exist.

The characteristics of autopoietic systems Maturana gives are as follows:

1. Autonomy by integration of various changes into the maintenance of their organization,
2. Individuality independent of mutual actions between them and external observers by repeatedly reproducing and maintaining the organization,
3. Self–determination of the boundary of the system through the self–reproduction processes,
4. Absence of input and output in the system by the fact that changes by any stimulus are subordinate to the maintenance of the organization which specifies the machine.

However, there has been still few mathematically strict models that represent autopoiesis. In [4, 5], we discussed the difficulty of interpreting autopoiesis within system theories using state spaces and problems of some models proposed for representing autopoiesis. The aim in this paper is to clarify whether autopoiesis can really be represented within more abstract mathematical frameworks by introducing the theory of category [9], one of the most abstract algebraic structure representing relations between components. The focus is the concept of "closure under efficient cause" in "relational biology" by Rosen [6].

In relational analysis, a system is regarded as a network that consists of components having functions. Rosen compared machine systems with living systems to clarify the difference between them, based on the relationship among components through entailment [6]. In other words, he focused his attention on where the function of each component results from in the sense of Aristotle's four causal categories, that is, material cause, efficient cause, formal cause, and final cause. As a result, Rosen claimed that a material system is an organism if and only if it is closed to efficient causation. In this paper, we consider that closure under entailment or production is a necessary condition for a system to be autopoietic because the components reproduce themselves in the system. Then, we give a system closed under entailment in a category theoretic framework.

2 Systems Closed under Entailment in a Category Theoretic Framework

In this paper, we assume that a category $\mathcal{C}$ has a final object 1 and product object $A \times B$ for any pair of objects A and B. The category of all sets is an example of this category. Moreover, we describe the set of morphisms from A to B as $H_{\mathcal{C}}(A, B)$ for any pair of objects A and B. An element of $H_{\mathcal{C}}(1, X)$ is called a morphic point on X. For a morphism $f \in H_{\mathcal{C}}(X, X)$ and a morphic point x on X, x is called a fixed point of f iff $f \circ x = x$ ($\circ$ means composition of morphisms) [8]. Morphic points and fixed points are respectively abstraction of elements of a set and fixed points of maps in the category of sets.

When there exists the power object Y^X for objects X and Y (that is, the functor $\cdot \times X$ on $\mathcal{C}$ has the right adjoint functor $\cdot^X$ for X), note that there is a natural one–to–one correspondence between $H_{\mathcal{C}}(Z \times X, Y)$ and $H_{\mathcal{C}}(Z, Y^X)$ for any objects X, Y, Z satisfying the diagram in the left figure of Fig. 1. Thus, there is a natural one–to–one corrspondence between morphic points on Y^X and morphisms from X to Y satisfying the diagram in the right figure of Fig. 1.

One of the easist methods for representing the self–reproductive aspect of autopoiesis is considered to assume that components in a system are not only operands but also operators [2]. Thus, we assume that there is an isomorphism from the space of operands to the space of operators, that is, an object X with powers and an isomorphism $f : X \simeq X^X$ in $\mathcal{C}$. Then, there uniquely exists a morphic point p on $(X^X)^X$ corresponding to f in the above sense. Since the morphism from X^X to $(X^X)^X$ entailed by the functor $\cdot^X$, f^X, is also isomorphic, there uniquely exists a morphic point q on X^X such that $f^X \circ q = p$. We can consider that p and q entail each other by f^X. Furthermore, there uniquely exists a morphic point x on X such that $f \circ x = q$. Since we can consider that x and q entail each other by f, and f and p entail each other by the

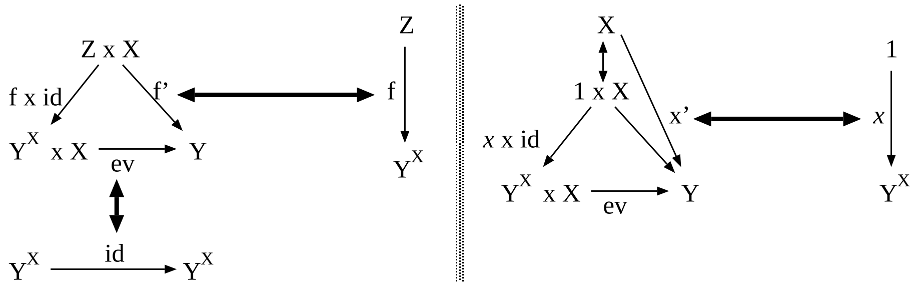

Fig. 1. Natural One–To–One Correspondence between $H_{\mathcal{C}}(Z \times X, Y)$ and $H_{\mathcal{C}}(Z, Y^X)$

natural correspondence, the system consisting of x, q, p, f, and f^X is completely closed under entailment. Moreover, if a set S of morphic points on X is fixed by $q' : X \to X$ naturally corresponding to q, that is, $\forall x' \in S\ q' \circ x' \in S$ and $\exists x'' \in S\ s.t.,\ q' \circ x'' = x'$, we can consider that S entails itself by f (the existence of these sets is guaranteed by Theorem 1 in [8], that is, the fact that q' has fixed points by f as a labelling of X^X by X).

Fig. 2 shows the diagrams of this completely closed system and its hyper-digraph [1] representing the relationship on entailment between components (a thick line starting from a thin line means that the components connected by the thin line entails the component at which the thick line ends). Thus, one isomorphism from X to X^X generates one completely closed system.

3 Conclusion and Discussion

We proposed completely closed systems under entailment in Sec. 2 by assuming the existence of an isomorphism between an object and its power object. Although we cannot do in this paper due to the page limit, we can provide another type of closed systems by assuming similar conditions and abstracting Rosen's (M,R) systems [7].

Although we need to consider some future problems such as coupling of these closed systems, the most important problem is the condition of the category used for constructing closed system. Although we required that operands coincide with operators ($X \simeq X^X$), this condition is difficult to be satisfied in the naive set theory. Although Soto–Andrade and Varela provided a category satisfying this condition (the category of partially ordered sets and continuous monotone maps with special conditions)[8], this category is very special. Furthermore, Rosen showed that systems closed under efficient cause cannot be described with their states because they lead to infinite regress [6]. We have still not clarified whether the existence of an isomorphism between an object and its power object is a sufficient condition for a system to be closed under entailment in the category theoretic framework. If these closed systems can exist only in special categories

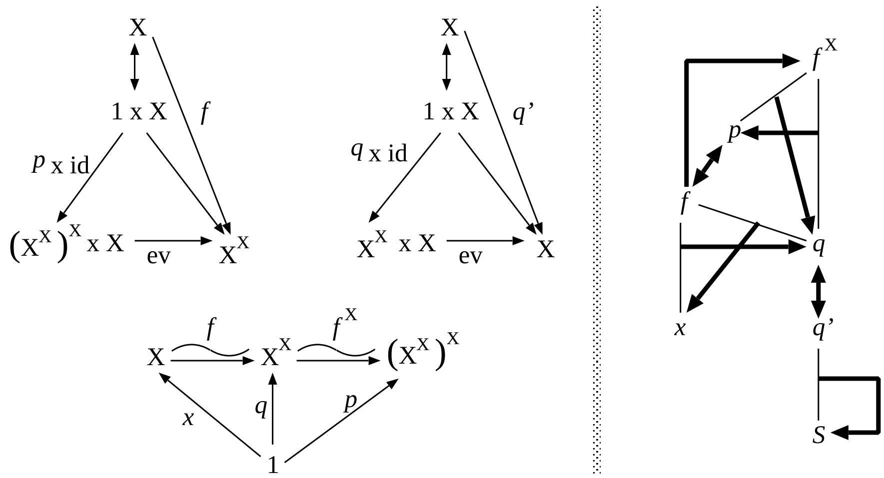

Fig. 2. Diagrams of a Completely Closed System and Its Hyperdigraph on Entailment

not observable in the conventional sense, however, autopoiesis may be hard to be a general theory of a variety of systems.

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