Category Theoretical Formalization of Autopoiesis from Perspective of Distinction between Organization and Structure

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Abstract

Since the concept of autopoiesis was proposed as a model of minimal living systems by Maturana and Varela, and applied to social systems by Luhmann, there has been only a few mathematically strict models to represent the characteristics of it because of its difficulty for interpretation. In order to explore the validity of this concept, this paper proposes a more general formal description of autopoiesis based on the theory of category. This paper focuses on the distinction of organizations and structures, and then discusses its implications and problems on formalization of autopoiesis.

1 Introduction

Autopoiesis gives a framework in which a system exists as an organism through physical and chemical processes, based on the assumption that organisms are machinery [7]. According to the original definition of it by Maturana and Varela, an autopoietic system is one that continuously produces the components that specify it, while at the same time realizing itself to be a concrete unity in space and time; this makes the network of production of components possible. An autopoietic system is organized as a network of processes of production of components, where these components:

1. continuously regenerate and realize the network that produces them, and
2. constitute the system as a distinguishable unity in the domain in which they exist.

The characteristics of autopoietic systems Maturana gives are as follows:

Autonomy: Autopoietic machinery integrates various changes into the maintenance of its organization. A car, the representative example of non–autopoietic systems, does not have any autonomy.

Individuality: Autopoietic machinery has its identity independent of mutual actions between it and external observers, by repeatedly reproducing and maintaining the organization. The identity of a non–autopoietic system is dependent on external observers and such a system does not have any individuality.

Self–Determination of the Boundary of the System: Autopoietic machinery determines its boundary through the self–reproduction processes. Since the boundaries of non–autopoietic systems are determined by external observers, self–determination of the boundaries does not apply to them.

Absence of Input and Output in the System: Even if a stimulus independent of an autopoietic machine causes continuous changes in the machine, these changes are subordinate to the maintenance of the organization which specifies the machine. Thus, the relation between the stimulus and the changes lies in the area of observation, and not in the organization.

This system theory has been applied to a variety of fields including sociology [5]. However, there has been only a few mathematically strict models to represent the characteristics of it because of its difficulty for interpretation. McMullin has studied a computational model of autopoiesis as 2–D biological cells [9]. Bourgine and Stewart proposed a mathematical formalization of autopoiesis as random dynamical systems and explored the relationships between autopoiesis and cognitive systems [2]. Letelier, et. al., suggested the relationships between autopoiesis and metabolism–repair systems [6], which is an abstract mathematical model of biological cells proposed by Rosen [13]. Nomura also proposed a mathematical model of autopoiesis based on Rosen's system [10]. These models vary from abstract algebraic formalization to computational models based on artificial chemistry.

In order to explore the validity of autopoiesis more deeply, this paper reconsiders necessary conditions for modeling characteristics of autopoiesis based on Kawamoto's theory [4], and discusses a problem of the above existing models of autopoiesis. On these consideration and discussion, this paper uses the category theoretic formalization of autopoiesis, which was proposed by Nomura [11, 12] to clarify whether autopoiesis can really be represented within the conventional mathematical frameworks.

2 Distinction between Organization and Structure

In Japan, Kawamoto has continued his own development of autopoiesis [4]. Kawamoto's theory focuses on circular relations of components and the network of production processes of components.

Kawamoto's important claims are as follows: an autopoietic system is a network consisting of relations between production processes of components. This network produces components of the system, and the components exist in physical spaces. Then, the system exists only if the components reproduce the network of production processes. The structure of the system is a realization of the system through the operation of the system in the physical space, and the organization of the system is a form of the network. The organization is functionally specified, although the structure is realized in the physical space.

The above claims by Kawamoto have an important implication. The organization of a system differs from the structure since they exist in different levels. This distinction is mentioned in Maturana and Varela's original literature [8]. Figure 1 shows this aspect. The distinction between organizations and structures in an autopoietic system can be interpreted as a distinction between categories on which the organization and structure of a autopoietic system are defined in a mathematical formalization of it.

This distinction is suggested from another formal perspective.

Rosen compared machine systems with living systems to clarify the difference between them,

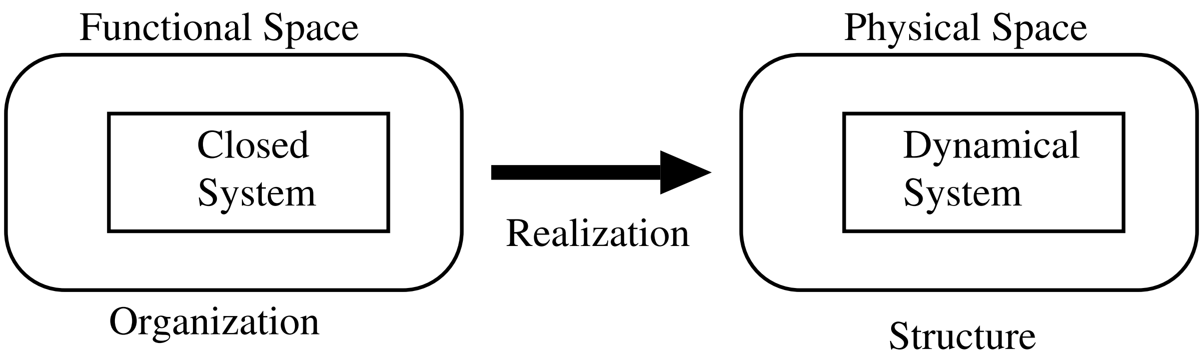

Figure 1: Aspect of Autopoiesis based on Distinction between Organization and Structure

based on the relationship among components through entailment [13]. In other words, he focused his attention on where the function of each component results from in the sense of Aristotle's four causal categories, that is, material cause, efficient cause, formal cause, and final cause. As a result, Rosen claimed that a material system is an organism if and only if it is closed to efficient causation. Furthermore, Rosen suggested that systems closed under efficient cause cannot be described with their states because they lead to infinite regress [13].

Nomura proposed a category theoretic formalization of autopoiesis under the assumption that closure under entailment or production is a necessary condition for a system to be autopoietic because the components reproduce themselves in the system [11, 12]. Although this formalization showed the possibility of constructing systems closed under entailment in specific categories, these categories had to satisfy the condition that operands coincide with operators ($X \simeq X^X$). Although Soto–Andrade and Varela provided a category satisfying this condition (the category of partially ordered sets and continuous monotone maps with special conditions), this category is very special [14].

The above two works have an important implication. If circular relations between components and their production process network are closed under entailment, this closedness may be hard to be formalized in general state spaces. On the other hand, the structure of an autopoietic system must be realized in a state space as a physical one. These implications suggest the distinction between organizations and structures in formalization of autopoiesis.

However, there is no general model of autopoiesis reflecting this distinction. The existing computational models and dynamical system models are defined on state spaces specific for them. In other words, they specify structures of systems as relations between elements of the systems, and have no explicit formalization of the organizations.

On the other hand, Nomura's category theoretic model [11, 12] represents only the aspect of closedness in organizations, and lacks the structures in autopoiesis. The next section proposes a category theoretic formalization of autopoiesis involving the distinction between organizations and structures, by complementing this lack.

3 Category Theoretic Model of Distinction between Organizations and Structures

3.1 Theory of Category

Category theory is an algebraic framework to abstractly handle the collection of mathematical objects having some specific properties, such as “the collection of all groups”, “the collection of all sets”, “the collection of all topological spaces”, “the collection of differential manifolds”, and so on [1]. In this framework, an individual space or set is dealt with as an object, and a function or map from an object to another object is dealt with as a morphism corresponding to an arc between them. Thus, the inner structures of any object and morphism are reduced, and pure relations of morphisms between are focused on. This can make it possible to investigate what category of mathematical objects a specific relation between objects (for example, closed relations between objects and morphisms) is satisfied in.

In addition, category theory can deal with relations of categories themselves as functors. This can make it possible investigate relations between a specific category and general ones such as state spaces.

As mentioned in the previous section, the organization of an autopoietic system should be formalized as a network of components and production processes, closed under entailment. Then, the structure of the system should be realized in a state space. The proposal in this paper is that the organization is formalized in a specific category, the structure is formalized in the category of general state spaces, and realization from the organization to the structure in the autopoietic system is modeled by a functor between the categories.

3.2 Completely Closed Systems as Organizations

This paper shows a formalization using “completely closed systems” as an example of organization [11, 12]. Although there are other closed systems to be considered as organizations, this paper focuses on this simple type of systems to provide with easier interpretation.

We assume that an abstract category $\mathcal{C}$ has a final object 1 and product object $A \times B$ for any pair of object A and B. The category of all sets is an example of this category. Moreover, we describe the set of morphisms from A to B as $H_{\mathcal{C}}(A, B)$ for any pair of objects A and B. A element of $H_{\mathcal{C}}(1, X)$ is called a morphic point on X. For a morphism $f \in H_{\mathcal{C}}(X, X)$ and a morphic point x on X, x is called a fixed point of f iff $f \circ x = x$ ($\circ$ means composition of morphisms) [14]. Morphic points and fixed points are respectively abstraction of elements of a set and fixed points of maps in the category of sets.

When there exists the power object Y^X for objects X and Y (that is, the functor $\cdot \times X$ on $\mathcal{C}$ has the right adjoint functor $\cdot^X$ for X), note that there is a natural one–to–one correspondence between $H_{\mathcal{C}}(Z \times X, Y)$ and $H_{\mathcal{C}}(Z, Y^X)$ for any objects X, Y, Z satisfying the diagram in the left half of figure 2. Thus, there is a natural one–to–one correspondence between morphic points on Y^X and morphisms from X to Y satisfying the diagram in the right half of figure 2 [15].

When components in a system are not only operands but also operators, the easiest method for representing this aspect is the assumption of existence of an isomorphism from the space of operands to the space of operators [3].

Now, we assume an object X with powers and an isomorphism $f : X \simeq X^X$ in $\mathcal{C}$. Then, there uniquely exists a morphic point p on $(X^X)^X$ corresponding to f in the above sense, that

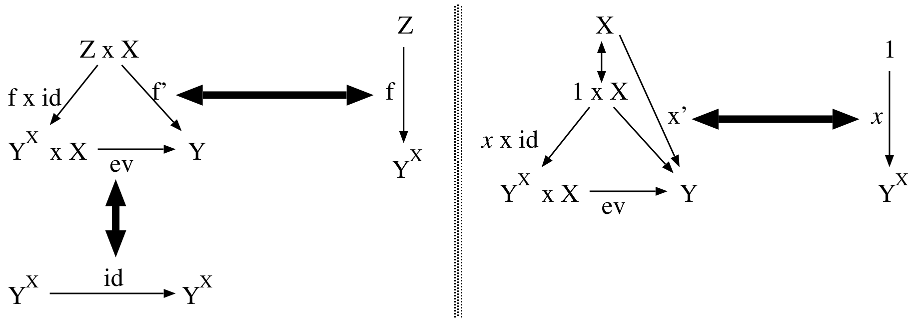

Figure 2: Natural One–To–One Correspondence between $H_{\mathcal{C}}(Z \times X, Y)$ and $H_{\mathcal{C}}(Z, Y^X)$

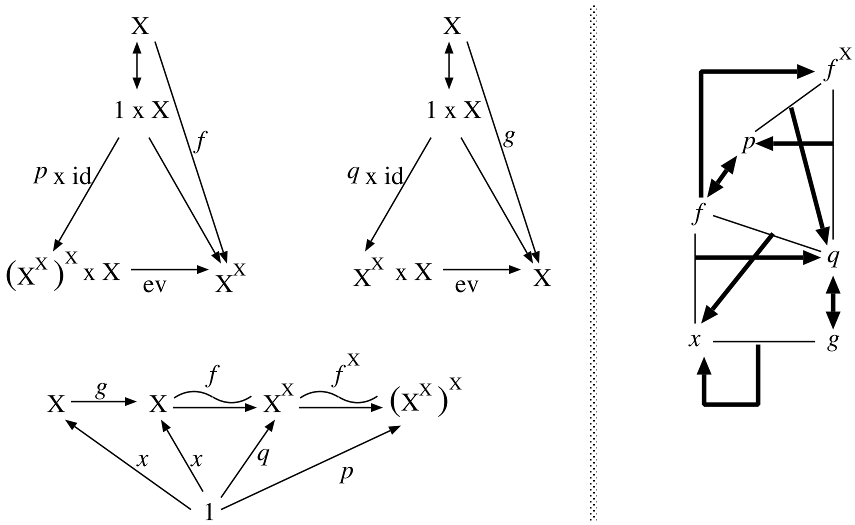

Figure 3: The Diagrams of a Completely Closed System and the Entailment Relations

is , $p' = f$. Since the morphism from X^X to $(X^X)^X$ entailed by the functor $\cdot^X$, f^X, is also isomorphic, there uniquely exists a morphic point q on X^X such that $f^X \circ q = p$. We can consider that p and q entail each other by f^X. Furthermore, there uniquely exists a morphic point x on X such that $f \circ x = q$ because f is isomorphic. Since we can consider that x and q entail each other by f, and f and p entail each other by the natural correspondence, the system consisting of x, q, p, f, and f^X is completely closed under entailment. Moreover, if x is a fixed point of $g : X \to X$ naturally corresponding to q, that is, $g \circ x = x$, we can consider that x entails itself by g. Figure 3 shows the diagrams of this completely closed system and the entailment relations.

3.3 Structures Induced by Completely Closed Systems

Here, we consider the formalization of structures induced by the organization mentioned in the previous section as follows.

We assume a family of categories $\{C_\lambda\}_\lambda$, and that each C_λ includes a completely closed system $\{X_\lambda, x_\lambda \in H_{C_\lambda}(1, X_\lambda), f_\lambda \in H_{C_\lambda}(X_\lambda, {X_\lambda}^{X_\lambda}), q_\lambda \in H_{C_\lambda}(1, {X_\lambda}^{X_\lambda}), p_\lambda \in H_{C_\lambda}(1, \left({X_\lambda}^{X_\lambda}\right)^{X_\lambda}), g_\lambda \in H_{C_\lambda}(X_\lambda, X_\lambda)\}$. Moreover, we assume another category D. Here, it is assumed that D is the category consisting of state spaces and maps between them, or its subcategory.

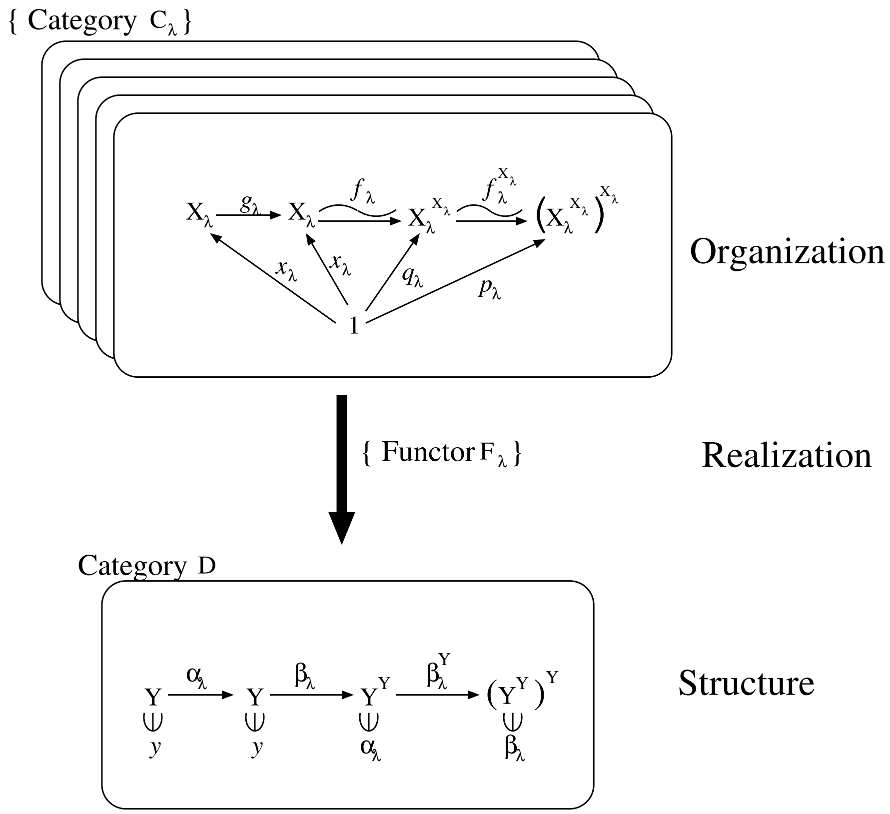

Figure 4: Organization of an Autopoietic System and Its Realized Structure

Now, we assume a family of functors $\{F_\lambda : C_\lambda \to D\}_\lambda$ such that $F_\lambda(1_{C_\lambda}) = 1_D$, $Y = F_\lambda(X_\lambda)$, $y = F_\lambda(x_\lambda) \in H_D(1, Y) = Y$ for all λ, and $\alpha_\lambda = F_\lambda(g_\lambda) \in H_D(Y, Y) = H(1, Y^Y) = Y^Y$ for any λ (here, we can regard $H_D(1, Y) = Y$ and $H_D(Z, Y) = H_D(1, Y^Z) = Y^Z$ since D is the category of state spaces). Note that $\alpha_\lambda(y) = \alpha_\lambda \circ y = F_\lambda(g_\lambda) \circ F_\lambda(x_\lambda) = F_\lambda(g_\lambda \circ x_\lambda) = F_\lambda(x_\lambda) = y$ for any λ, that is, y is a fixed point of α_λ.

We propose that the family of the completely closed systems $\{\{X_\lambda, x_\lambda, f_\lambda, q_\lambda, p_\lambda, g_\lambda\}\}_\lambda$ is an organization of an autopoietic system and $\{Y, y, \{\alpha_\lambda\}_\lambda\}$ is its structure realized on the category D through the family of the functors $\{F_\lambda\}$ if for any λ one of the following conditions is satisfied:

1. $^\exists\beta_\lambda \in H_D(Y, Y^Y) = H_D\left(1, (Y^Y)^Y\right) = (Y^Y)^Y$ $s.t.$, $\beta_\lambda(y) = \alpha_\lambda$, ${\beta_\lambda}^Y(\alpha_\lambda) = \beta_\lambda$
2. $^\exists\beta_\lambda \in H_D(Y, Y^Y)$, λ_1, λ_2, and $\beta_{\lambda_2} \in H_D(Y, Y^Y)$ $s.t.$, $\beta_\lambda(y) = \alpha_{\lambda_1}$, $\beta_\lambda = {\beta_{\lambda_2}}^Y(\alpha_{\lambda_2})$

The above relationship between the organization and structure represents the aspect that the structure is entailed repeatedly within the organization. Figure 4 shows these organization and realized structure.

4 Discussion

This section discusses some implications of the formalization of autopoiesis proposed in the previous section, and some problems of it.

4.1 Implications

The proposed formalization of autopoiesis explicitly represents a distinction between organizations and structures. The following four facts can be implied from this representation.

First, the organization is static and closed under entailment between morphic points and morphisms. This implies the aspect of autopoiesis that the network consisting of relations between production processes of components is reproduced by the components.

Second, by distinguishing the category on which the structure is realized from the categories on which the organization is defined, a kind of dynamics in the structure is implied. On this dynamics, a part of the structure α_λ is dealt with. The first condition of structure in the previous section means that α_λ is dynamically maintained and the structure is closed by the existence of β_λ. The second condition of structure in the previous section means that α_λ is dynamically changed within the organization, that is, change of the structure under the unique organization.

Third, by introducing realization as a family of functors from the categories of organization to the category of state spaces, non–uniqueness of structures for the organization is represented. In other words, for the same organization $\{\{X_\lambda, x_\lambda, f_\lambda, q_\lambda, p_\lambda, g_\lambda\}\}_\lambda$, the existence of another structure $\{Y', y', \{\alpha'_\lambda\}_\lambda\}$ and its realization $\{F'_\lambda\}$ are implied. This suggests that a structure of autopoiesis having an organization based some physical materials can be realized based on other materials.

Fourth, the formalization in the paper uses completely closed systems as a simpler example of organization. Of course, closed systems of organizations are not limited to completely closed systems [12]. Thus, the proposed formalization implies a variety of organizations, structures, and realization.

4.2 Problems

On the other hand, the proposed formalization of autopoiesis has the following problems.

The proposed formalization assumes that organizations are formalized on categories permitting the existence of an isomorphism from the space of operands to the space of operators, in prior to state spaces on which the structures are realized. Then, change of the structures is fixed within the organizations. This is anticipation of the issues in a sense.

This is critical when we consider whether a system can be identified as autopoiesis by observers who can only see the structure on a state space. In order for the observers to be able to identify the system as autopoiesis, they must be able to find the organizations that cannot be formalized on the state space, and the categories of functional spaces on which the organizations are closed in the sense that the network consisting of relations between production processes of components is reproduced by the components. When these observers assume the organizations based on only the structures, however, there is arbitrariness in this assumption since the proposed model does not include any specification of organizations from structures. In this sense, the proposed formalization of autopoiesis may not be autopoiesis itself but just a cognitive model of the way in which these observers identify the system as autopoiesis.

This critical problem is caused by explicit distinction of organizations and structures, and closedness of organizations. As far as closedness of organizations is assumed, organizations are hard to be found in state spaces on which structures are realized. Thus, another category in an abstract level is necessary. As one of ways to refine the proposed formalization, we consider its application to the existing computational and dynamical systems models of autopoiesis, that is, investigation of categories of organization assuming these models as structures. By this application, we can find whether the proposed formalization can discriminate between autopoietic and non–autopoietic systems.

5 Summary

This paper focused on the distinction of organizations and structures in autopoiesis reconsidering necessary conditions for modeling characteristics of autopoiesis based on Kawamoto's theory [4]. Then, a general formalization of autopoesis based on category theory was proposed while explicitly representing the distinction of organizations and structures. In addition, some implications and problems were discussed.

As an important future work, we consider application of the proposed framework to real systems including biological, mental, and social systems. This can allow us to investigate whether the proposed framework is useful to clarify the difference between autopoietic and non–autopoietic systems at an abstract mathematical level.

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