Category Theoretical Distinction between Autopoiesis and (M,R) Systems

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Abstract. Some research works have mentioned the similarity of autopoiesis with (M,R) systems proposed by Rosen, from the perspective of closedness of the systems. However, there are some difference between the aspects of closedness required for autopoiesis and (M,R) systems. This paper aims at clarifying these differences to investigate the possibility of algebraic description of living systems, based on category theoretic frameworks.

1 Introduction

Autopoiesis gives a framework in which a system exists as an organism through physical and chemical processes, based on the assumption that organisms are machinery [9]. This system theory has been applied to a variety of fields including sociology [7]. However, there has been only a few mathematically strict models to represent the characteristics of it because of its difficulty for interpretation. McMullin has studied a computational model of autopoiesis as 2–D biological cells [11]. Bourgine and Stewart proposed a mathematical formalization of autopoiesis as random dynamical systems and explored the relationships between autopoiesis and cognitive systems [2].

On the other hand, some research works have mentioned the similarity of autopoiesis with metabolism–repair ((M,R)) systems, which is an abstract mathematical model of biological cells proposed by Rosen [16], from the perspective of closedness of the systems. Letelier et al, [8] reviewed (M,R) systems and provided with their algebraic example while suggesting the relationship with autopoiesis. Chemero and Turvey [3] proposed a system formalization based on hyperset theory and found a similarity between (M,R) systems and autopoiesis on closedness. Nomura [12, 13] also proposed some mathematical models of autopoiesis while connecting between closedness of autopoiesis and (M,R) systems.

When autopoiesis and (M,R) systems are compared in the abstract level based on category theory, however, there are some difference between the aspects of closedness required for autopoiesis and (M,R) systems. To explore algebraic models of living systems, this paper clarifies these differences and reconsiders necessary conditions for modeling characteristics of autopoiesis. On these consideration and discussion, this paper uses the category theoretic formalization of autopoiesis, which was proposed by Nomura [12–14] to clarify whether autopoiesis can really be represented within the conventional mathematical frameworks.

2 Autopoiesis and (M,R) Systems

2.1 Autopoiesis

An autopoietic system is organized as a network of processes of production of components, where these components:

1. continuously regenerate and realize the network that produces them, and
2. constitute the system as a distinguishable unity in the domain in which they exist.

The characteristics of autopoietic systems Maturana gives are as follows:

Autonomy: Autopoietic machinery integrates various changes into the maintenance of its organization. A car, the representative example of non–autopoietic systems, does not have any autonomy.

Individuality: Autopoietic machinery has its identity independent of mutual actions between it and external observers, by repeatedly reproducing and maintaining the organization. The identity of a non–autopoietic system is dependent on external observers and such a system does not have any individuality.

Self–Determination of the Boundary of the System: Autopoietic machinery determines its boundary through the self–reproduction processes. Since the boundaries of non–autopoietic systems are determined by external observers, self–determination of the boundaries does not apply to them.

Absence of Input and Output in the System: Even if a stimulus independent of an autopoietic machine causes continuous changes in the machine, these changes are subordinate to the maintenance of the organization which specifies the machine. Thus, the relation between the stimulus and the changes lies in the area of observation, and not in the organization.

In Japan, Hideo Kawamoto has continued his own development of autopoiesis [6]. He designated the properties of autopoiesis by comparison with conventional system theories. In particular, he focuses on the fourth item among the above characteristics of autopoiesis, i.e., absence of input and output in the system.

When we consider the "absence of input and output", important is the view where the system is understood based on the production processes. Kawamoto claims the following: the view of the relation between inputs and outputs in the system is one from external observers and it does not clarify the organization or the operation of the production in the system. A living cell only reproduces its components and does not produce the components while adjusting itself according to the relation between itself and oxygen in the air. Although the density of oxygen affects the production processes, external observers decide the influence and the cell does not. As long as the system is grasped from an internal view of the cell, the system does not have any "inputs and outputs".

From the above perspective, Kawamoto's theory focuses on circular relations of components and the network of production processes of components. His important claims are as follows: an autopoietic system is a network consisting of relations between production processes of components. This network produces components of the system, and the components exist in physical spaces. Then, the system exists only if the components reproduce the network of production processes. The structure of the system is a realization of the system through the operation of the system in the physical space, and the organization of the system is a form of the network. The organization is functionally specified and closed, although the structure is realized in the physical space.

2.2 (M,R) Systems and Closure under Entailment

In relational analysis, a system is regarded as a network that consists of components having functions. Rosen compared machine systems with living systems to clarify the difference between them, based on the relationship among components through entailment [16]. In other words, he focused his attention on where the function of each component results from in the sense of Aristotle's four causal categories, that is, material cause, efficient cause, formal cause, and final cause. As a result, Rosen claimed that a material system is an organism if and only if it is closed to efficient causation. Furthermore, Rosen suggested that systems closed under efficient cause cannot be described with their states because they lead to infinite regress.

(M,R) systems [15] satisfy closure under efficient cause. This system model maintains its metabolic activity through inputs from environments and repair activity. The simplest (M,R) systems represent the above aspect in the following diagram and the left half in figure 1.

$$A \xrightarrow{f} B \xrightarrow{\phi_f} H(A,B) \xrightarrow{\Phi_f} H(B,H(A,B)) \tag{1}$$

Here, A is a set of inputs from an environment to the system, B is a set of outputs from the system to the environment, f is a component of the system represented as a map from A to B, and ϕ_f is the repair component of f as a map from B to $H(A,B)$ ($H(X,Y)$ is the set of all maps from a set X to a set Y). In biological cells, f corresponds to the metabolism, and ϕ_f to the repair. If $\phi_f(b) = f$ ($b = f(a)$) is satisfied for the input $a \in A$, we can say that the system maintains itself. In addition, Φ_f can be constructed by the preceding (M, R) system in the following way: For a and b such that $b = f(a)$ and $\phi_f(b) = f$, if $\hat{b} : H(B,H(A,B)) \to H(A,B)$ ($\hat{b}(\phi)(a') = \phi(b)(a')$ ($\phi \in H(B,H(A,B)), a' \in A$)) has the inverse map $\hat{b}^{-1}$, it is easily proved that $\hat{b}^{-1}(f) = \phi_f$. Thus, we can set $\Phi_f = \hat{b}^{-1}$. The right half in figure 1 shows the aspect that the components except for a are closed under entailment.

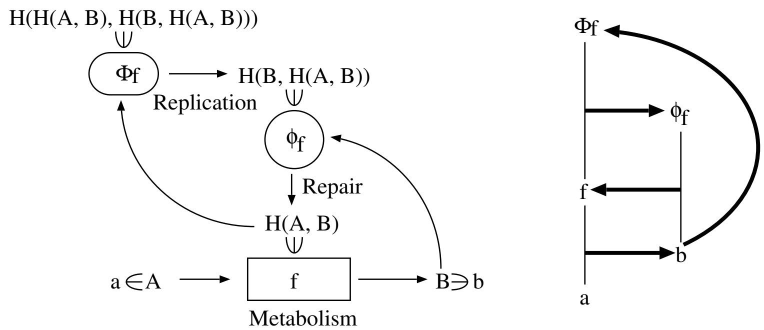

Fig. 1. A (M,R) System and Its Entailment Relation

3 Category Theoretical Formalization of Autopoiesis and (M,R) Systems

Category theory is an algebraic framework to abstractly handle the collection of mathematical objects having some specific properties, such as "the collection of all groups", "the collection of all sets", "the collection of all topological spaces", "the collection of differential manifolds", and so on [1]. In this framework, an individual space or set is dealt with as an object, and a function or map from an object to another object is dealt with as a morphism corresponding to an arc between them. Thus, the inner structures of any object and morphism are reduced, and pure relations of morphisms between objects are focused on. This can make it possible to investigate what category of mathematical objects a specific relation between objects (for example, closed relations between objects and morphisms) is satisfied in. In addition, category theory can deal with relations of categories themselves as functors. This can make it possible investigate relations between a specific category and general ones such as state spaces.

In this paper, we assume that an abstract category $\mathcal{C}$ has a final object 1 and product object $A \times B$ for any pair of object A and B. The category of all sets is an example of this category. Moreover, we describe the set of morphisms from A to B as $H_{\mathcal{C}}(A, B)$ for any pair of objects A and B. A element of $H_{\mathcal{C}}(1, X)$ is called a morphic point on X. For a morphism $f \in H_{\mathcal{C}}(X, X)$ and a morphic point x on X, x is called a fixed point of f iff $f \circ x = x$ ($\circ$ means concatenation of morphisms) [17]. Morphic points and fixed points are respectively abstraction of elements of a set and fixed points of maps in the category of sets. This abstraction is useful when our discussion is extended to categories of which objects and morphisms are not assumed to be sets and their maps, such as the Lindenbaum category of which objects and morphisms are the constants and equivalence classes of formulas of a formal theory [17].

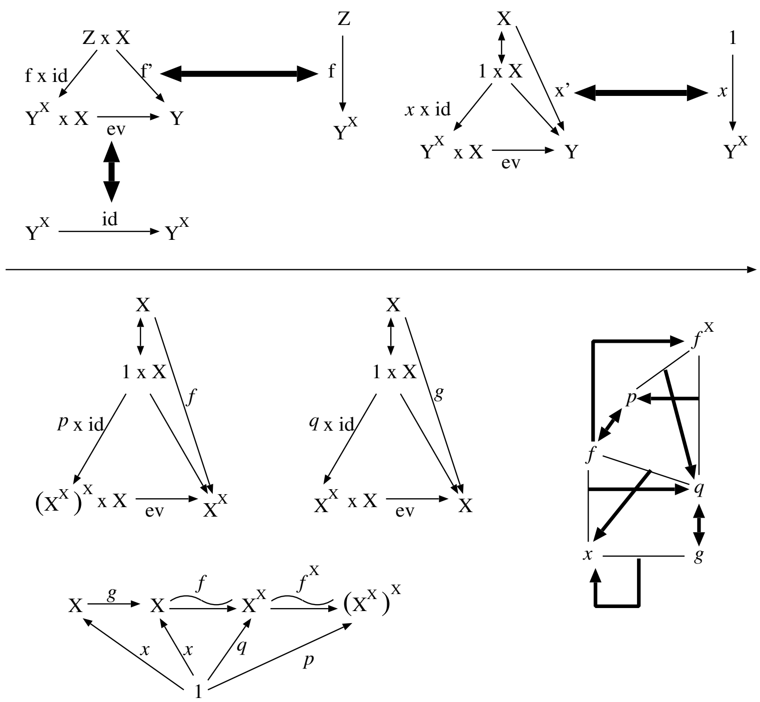

Fig. 2. The Diagrams of a Completely Closed System and the Entailment Relations based on Natural One–To–One Correspondence

3.1 Category Theoretical Formalization of Autopoiesis

The fact that the components reproduce themselves in a system implies that the components are not only operands but also operators. The easiest method for realizing this implication is the assumption of existence of an isomorphism from the space of operands to the space of operators [5]. Under this assumption, Nomura [12, 13] proposed completely closed systems under entailment between the components.

When there exists the power object Y^X for objects X and Y (that is, the functor $\cdot \times X$ on $\mathcal{C}$ has the right adjoint functor $\cdot^X$ for X), note that there is a natural one–to–one correspondence between $H_{\mathcal{C}}(Z \times X, Y)$ and $H_{\mathcal{C}}(Z, Y^X)$ for any objects X, Y, Z satisfying the diagram in the upper figure of figure 2 [18]. Thus, there is a natural one–to–one correspondence between morphic points on Y^X and morphisms from X to Y satisfying the diagram in the lower figure of figure 2. By using the above property, we can construct completely closed systems as follows.

Now, we assume an object X with powers and an isomorphism $f : X \simeq X^X$ in $\mathcal{C}$. Then, there uniquely exists a morphic point p on $(X^X)^X$ corresponding to f in the above sense, that is , $p' = f$. Since the morphism from X^X to $(X^X)^X$

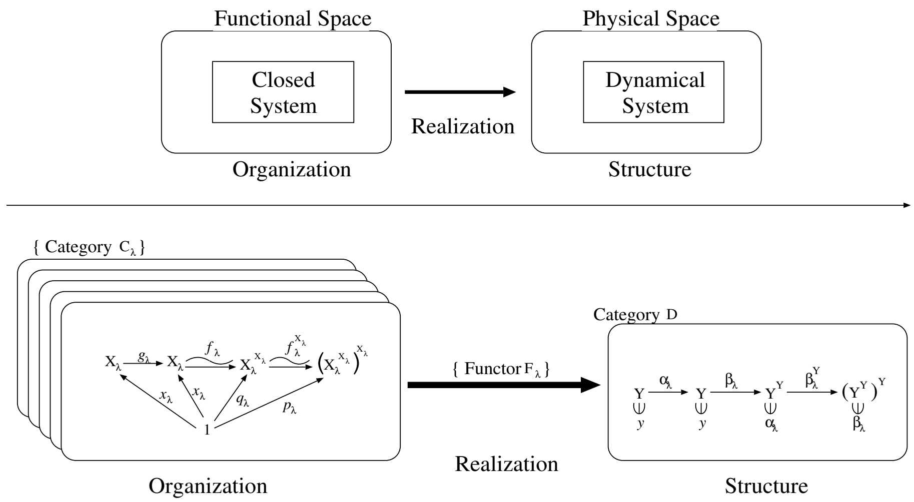

Fig. 3. Aspect of Autopoiesis based on Distinction between Organization and Structure, and Its Category Theoretical Formalization

entailed by the functor $\cdot^X$, f^X, is also isomorphic, there uniquely exists a morphic point q on X^X such that $f^X \circ q = p$. We can consider that p and q entail each other by f^X. Furthermore, there uniquely exists a morphic point x on X such that $f \circ x = q$ because f is isomorphic. Since we can consider that x and q entail each other by f, and f and p entail each other by the natural correspondence, the system consisting of x, q, p, f, and f^X is completely closed under entailment. Moreover, if x is a fixed point of $g : X \to X$ naturally corresponding to q, that is, $g \circ x = x$, we can consider that x entails itself by g. The lower figure of Figure 2 shows the diagrams of this completely closed system and the entailment relations.

Furthermore, Autopoiesis argues not only closedness of entailment between the components but also two levels of description. Kawamoto's claims mentioned in the previous section have an important implication. The organization of a system differs from the structure since they exist in different levels. This distinction is mentioned in Maturana and Varela's original literature [10]. The upper figure of Figure 3 shows this aspect. The distinction between organizations and structures in an autopoietic system can be interpreted as a distinction between categories on which the organization and structure of a autopoietic system are defined in a mathematical formalization of it.

This fact has an important implication. If circular relations between components and their production process network are closed under entailment, this closedness may be hard to be formalized in general category such as state spaces. On the other hand, the structure of an autopoietic system must be realized in a state space as a physical one. These implications suggest the distinction between organizations and structures in formalization of autopoiesis.

To represent the distinction between organizations and structures, Nomura [14] proposed a model in which the organization is formalized in a specific category, the structure is formalized in the category of general state spaces, and realization from the organization to the structure is represented by a functor between the categories. The lower figure of Figure 3 shows the model.

We assume a family of categories $\{C_\lambda\}_\lambda$, and that each C_λ includes a completely closed system $\{X_\lambda,\ x_\lambda \in H_{C_\lambda}(1, X_\lambda),\ f_\lambda \in H_{C_\lambda}(X_\lambda, {X_\lambda}^{X_\lambda}),\ q_\lambda \in H_{C_\lambda}(1, {X_\lambda}^{X_\lambda}),\ p_\lambda \in H_{C_\lambda}(1, \left({X_\lambda}^{X_\lambda}\right)^{X_\lambda}),\ g_\lambda \in H_{C_\lambda}(X_\lambda,\ X_\lambda)\}$. Moreover, we assume another category D. Here, it is assumed that D is the category consisting of state spaces and maps between them, or its subcategory.

Now, we assume a family of functors $\{F_\lambda : C_\lambda \to D\}_\lambda$ such that $F_\lambda(1_{C_\lambda}) = 1_D$, $Y = F_\lambda(X_\lambda)$, $y = F_\lambda(x_\lambda) \in H_D(1, Y) = Y$ for all λ, and $\alpha_\lambda = F_\lambda(g_\lambda) \in H_D(Y, Y) = H(1, Y^Y) = Y^Y$ for any λ (here, we can regard $H_D(1, Y) = Y$ and $H_D(Z, Y) = H_D(1, Y^Z) = Y^Z$ since D is the category of state spaces). Note that $\alpha_\lambda(y) = \alpha_\lambda \circ y = F_\lambda(g_\lambda) \circ F_\lambda(x_\lambda) = F_\lambda(g_\lambda \circ x_\lambda) = F_\lambda(x_\lambda) = y$ for any λ, that is, y is a fixed point of α_λ.

The family of the completely closed systems $\{\{X_\lambda, x_\lambda, f_\lambda, q_\lambda, p_\lambda, g_\lambda\}\}_\lambda$ is an organization of an autopoietic system and $\{Y, y, \{\alpha_\lambda\}_\lambda\}$ is its structure realized on the category D through the family of the functors $\{F_\lambda\}$ if for any λ one of the following conditions is satisfied:

1. $^\exists\beta_\lambda \in H_D(Y, Y^Y) = H_D\left(1, (Y^Y)^Y\right) = (Y^Y)^Y$ *s.t.*, $\beta_\lambda(y) = \alpha_\lambda$, ${\beta_\lambda}^Y(\alpha_\lambda) = \beta_\lambda$
2. $^\exists\beta_\lambda \in H_D(Y, Y^Y)$, λ_1, λ_2, and $\beta_{\lambda_2} \in H_D(Y, Y^Y)$ *s.t.*, $\beta_\lambda(y) = \alpha_{\lambda_1}$, $\beta_\lambda = {\beta_{\lambda_2}}^Y(\alpha_{\lambda_2})$

The above relationship between the organization and structure represents the aspect that the structure is entailed repeatedly within the organization.

3.2 Category Theoretically Described (M,R) Systems

As mentioned in the previous section, (M,R) systems are closed under entailment except for the input a. We can re–write the closed part of (M,R) systems as follows.

For objects X and Y in $\mathcal{C}$, we assume that X has powers. When a morphism $f : X \to Y$ and a morphic point x on X are given, we assume that x satisfies the following conditions:

$$\begin{aligned} &\exists\, G_x \in H_{\mathcal{C}}(Y^X, Y) \\ &\quad s.t.,\ G_x \circ z = z' \circ x \text{ for any } z \in H_{\mathcal{C}}(1, Y^X) \\ &\text{and } G_x \text{ has its inverse morphism } F_x \in H_{\mathcal{C}}(Y, Y^X) \end{aligned} \quad (2)$$

here, z' is the morphism from X to Y naturally corresponding to the morphic point z on Y^X. When $y = f \circ x$ and x_f is the morphic point on Y^X naturally

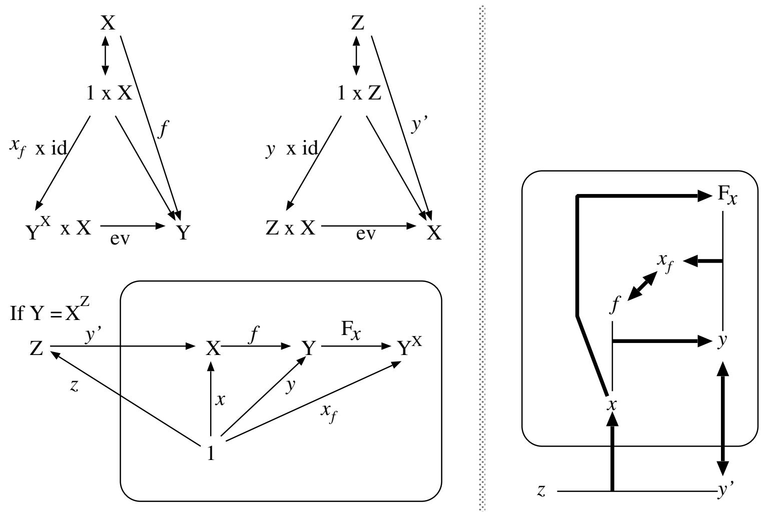

Fig. 4. The Diagrams of Category Theoretically Described (M,R) System and the Entailment Relations

corresponding to f $((x_f)' = f)$, we obtain $F_x \circ y = F_x \circ f \circ x = F_x \circ G_x \circ x_f = x_f$. Thus, x_f is entailed by y and F_x. If we regard F_x as entailed by x, then f, y, F_x, and x_f are entailed by themselves and x.

Furthermore, if there exist an object Z with powers and morphic point z on Z such that $Y = X^Z$ and $y' \circ z = x$, the system including y' and z represents the original (M,R) system.

Figure 4 shows the diagrams of this generalized (M,R) system and its entailment relations.

4 Differences between Autopoiesis and (M,R) Systems

It is considered that closure under entailment or production is a necessary condition for a system to be autopoietic because the components reproduce themselves in the system. In fact, the existing research works found the similarity of autopoiesis with (M,R) systems based on this closedness [8, 12, 13, 4, 3]. However, there are two points of difference between these systems. The first one is the difference on forms of closure.

The forms of closedness in completely closed systems as autopoiesis and (M,R) systems reveal the difference between them. In the completely closed system, the existence of isomorphism f between X and X^X determines complete closure under entailment without any condition. On the other hand, the closedness of the (M,R) system depends on whether one of the components x satisfies the condition represented in equation (2).

Moreover, there is also a difference on conditions of categories on which these systems are constructed. Although completely closed systems show the possibility of constructing systems closed under entailment in specific categories, these categories have to satisfy the condition that operands coincide with operators. Although Soto–Andrade and Varela [17] provided a category satisfying this condition (the category of partially ordered sets and continuous monotone maps with special conditions), this category is very special.

On the other hand, Rosen [16] argued based on category theoretic frameworks that systems closed under efficient cause like (M,R) systems cannot be described with their states because they lead to infinite regress. However, Chu and Ho [4] found that Rosen's proof for this argument was not complete since his proof assumes an implicit condition irrelevant from state space representation of systems. In fact, Letelier et al, [8] provided with an arithmetic example of a (M,R) system constructed within the category of finite groups. These facts imply the difference on types of categories required for autopoiesis and (M,R) systems.

The second difference between autopoiesis and (M,R) systems is based on distinction between organizations and structures. As mentioned in the previous section, autopoiesis requires this distinction. However, the form of (M,R) systems does not include the explicit distinction between closed organizations and structures realized in state spaces, and these concepts are confused.

5 Discussion and Conclusion

This paper suggested two differences between autopoiesis and (M,R) systems from the perspective of category theoretic formalization of them, the difference on forms of their closedness under entailment of the components and categories required for the closedness, and the existence of distinction between organizations and structures. However, the first difference depends on the assumption that completely closed systems are necessary conditions of autopoiesis, that is, the existence of an isomorphism from the space of operands to the space of operators is a necessary condition of autopoiesis. This proposition has not still been proved in a mathematically strict sense or sufficiently considered in a philosophical sense. We need to explore which mathematical conditions should be satisfied for formalization of autopoiesis.

Moreover, it should be sufficiently discussed what contribution the differences between autopoiesis and (M,R) systems suggested in the paper can provide with, in the sense of the above exploration of conditions of minimal living systems. As Chu and Ho [4] argue that Rosen's idea based on category theory can contribute to distinction between living and non–living systems, we also believe that category theoretical frameworks including Rosen's method will help us to bring us closer to an understanding of life systems.

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